μ = vector of control law coefficients
θ = vector of unmeasured process outputs
ϕ_{uu} = covariance matrix of the disturbances u
$\phi_{\theta\theta}$ = covariance matrix of the measurements θ
$\phi_{y\theta}$ = $E\{y\theta^T\}$, the cross correlation matrix of y with θ
$||[\cdot]||$ = norm of $[\cdot]$

Superscripts

$\wedge$ = estimated value
T = transpose

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Manuscript received February 2, 1977; revision received January 3, and accepted January 6, 1978.

Part II. The Structure and Dynamics of Inferential Control Systems

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The static estimator of part I is incorporated into a dynamic control system. The system structure minimizes output feedback and thereby achieves stability even in the event of significant modeling errors. This inherent stability permits the use of simple and economical lead-lag compensation for the estimator and controller portions of the system. The overall system response to disturbances is similar to that of feed-forward control systems.

The proposed control system structure is used to infer and control the overhead and bottoms product compositions of a simulated multicomponent distillation. Product compositions are inferred from selected stage temperature and process flow measurements. Inferential control system response to various disturbances is comparable or superior to that of a tuned composition feedback control system for both single product control and for simultaneus overhead and bottoms product control.

It is generally necessary to add some form of dynamic compensation to the static inferential control system obtained by the methods of Part I in order to obtain an operable control system. What is meant by an operable control system will vary from process to process, but commonly it will mean that the process lines out after a disturbance in less than three or four time constants without large excursions from the set point. Also implied in the idea of operability is the requirement that the control system be robust. That is, its response must not be seriously degraded by shifts in process operating conditions.

The block diagram structure for inferential control systems proposed in this work attempts to achieve an operable control system by assuring control system stability. The most critical transfer functions can be obtained from readily available plant data and/or simple plant tests. Owing to the inherent stability of the proposed control systems, the dynamic compensation of the estimator and controller need not be perfect, and simple lead-lag networks will usually suffice.

Inferential control systems can also be used in combination with feedforward and feedback control systems. Typical block diagrams for such configurations are presented later.

The dynamic behavior of inferential control systems with various types of modeling errors is discussed later. The important result in this section is that stability can almost always be assured by making sure that modeling errors are always in the appropriate direction.

We will present the results of applying inferential control, with ad hoc dynamic compensation to control the overhead and/or bottoms product compensation of a simulated multicomponent distillation column. Compositions are inferred from selected stage temperatures and process flows.

THE BLOCK DIAGRAM

A typical process block diagram is given to the right of the dotted line in Figure 1. All process inputs and outputs are vector quantities. Generally, the dimension of the product quality $y(s)$ and the control effort $m(s)$ vectors will be small, and frequently these variables are scalars. The number of secondary measurements $\theta(s)$ is under the control of the designer (see Part I), but economic considerations will usually limit the number of measurements to the order of 10 or less. The number of unmeasured disturbances $u(s)$ which enter a process varies widely with the process and can range from several hundred (for exam-

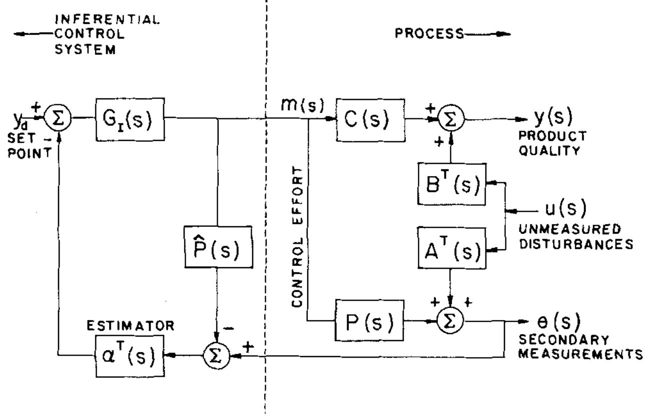

Fig. 1. Inferential control system.

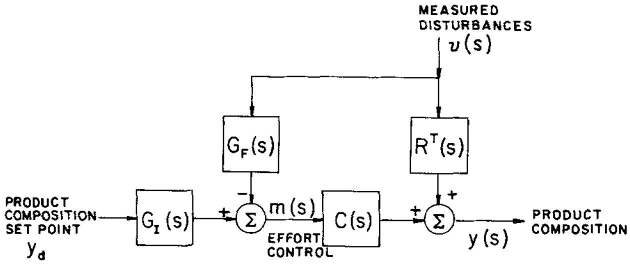

Fig. 2. Feedforward control system.

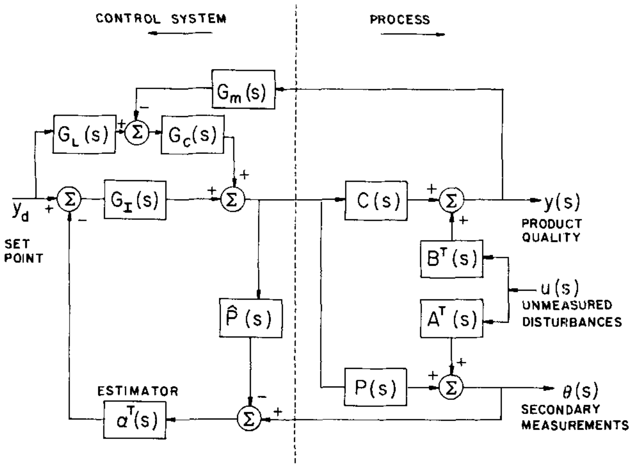

Fig. 3. Inferential control system with feedforward compensation.

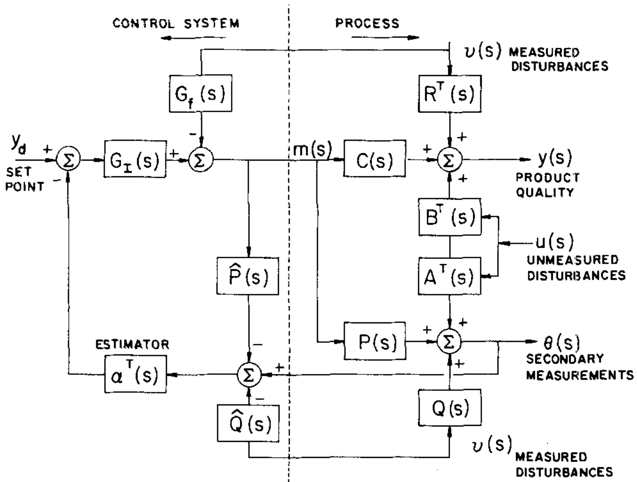

Fig. 4. Inferential control system with slow feedback control.

ple, a crude tower) to only one or two (for example, a dryer, Shinskey, 1973). Most frequently, the number of unmeasured disturbances will exceed the number of secondary measurements.

To the left of the process in Figure 1 is an inferential control system. It functions to counter the effect of the unmeasured disturbances $d(s)$ on the product quality. The control effort is based on an inference $\hat{d}(s)$ of $d(s)$. To isolate the effect of the unmeasured disturbances on the process outputs, the effect of all measured inputs (for example, the control effort) on the secondary measurements are subtracted from the measurement signal before it enters the estimator. If the transfer function matrix $\hat{P}(s)$ equals the true process transfer function matrix $P(s)$, then the signal which enters the estimator depends only on the disturbances $u(s)$. Further, there is no feedback of control effort $m(s)$ through the estimator $\alpha(s)$ and controller $G_I(s)$ to the secondary measurements $\theta(s)$.

The job of the estimator $\alpha(s)$ is to combine its input signals $A^T(s)u(s)$ in such a way as to obtain an estimate $\hat{d}(s)$ of the effect of the disturbances on the product quality. The controller $G_I(s)$ manipulates the control effort to produce the opposite effect $-\hat{d}(s)$ on the product quality. Cancellation is perfect if $d(s) = d(s)$, $G_I(s) = C^{-1}(s)$, and $C^{-1}(s)$ is stable. However, even if $\hat{d}(s) \neq d(s)$ and $G_I(s) \neq C^{-1}(s)$, the control system will be stable if $G_I(s)$ is stable and $\hat{P}(s)$ is a sufficiently good approximation to $P(s)$. Indeed, if $\hat{P}(s) = P(s)$, then the dynamic response of an inferential control system with imperfect estimation and control is similar to that of a feedforward control system with imperfect compensation. Both types of control systems respond only to disturbances and not to process outputs.

Implementation of the inferential control system of Figure 1 requires the implementation of $m(2n + m)$ transfer functions if all of the transfer function matrices $\alpha(s)$, $P(s)$, and $G_I(s)$ are full. [The dimension of the product quality vector $y(s)$ is m, and the dimension of the secondary measurement vector $\theta(s)$ is n.] Both the estimator matrix $\alpha(s)$ and the compensator matrix $P(s)$ have dimension $m \times n$. The controller $G_I(s)$ is $m \times m$. For most chemical processes, the quantity $m(2n + m)$ will be less than twenty-five. The implementation of twenty-five or so transfer functions is certainly not trivial, but it is economically feasible, provided that the transfer functions are simple lead-lag networks.

Figure 3 combines the inferential control system of Figure 1 with the feedforward control system of Figure 2. The combination of the two control systems would be linear were it not for the fact that the measured disturbances also affect the secondary measurements. Since the signal entering the estimator should reflect only changes in the unmeasured disturbances, the effect of the measured disturbances is removed by subtracting the signal $\hat{Q}(s)v(s)$ from the secondary measurements. The transfer function matrix $\hat{Q}(s)$ is our estimate of the effect of the measured disturbances on the secondary measurements.

Just as it is often desirable to add a slow feedback control system to a feedforward controller to eliminate steady state offset due to modeling errors, it will often be desirable to add a slow feedback control system to an inferential controller as shown in Figure 4 for the same reason. The feedback controller $G_c(s)$ will generally be a proportional plus integral controller which is slow by virtue of a large integral time constant and relatively low (perhaps zero) proportional gain. The measurement lag $G_m(s)$ is explicitly included in the feedback loop because this lag will often be more substantial than the process lags. (The measurement lags were omitted from the secondary measurement loop because these lags are generally small compared to the process lags.)

An unusual feature of the combined inferential and feedback control systems is that the set point $y_d(s)$ is used by both systems to achieve a rapid response to set point changes. The product quality responds rapidly to a change in the input to $G_I(s)$ since $C(s)G_I(s) \approx I$, the identity matrix. However, the product quality responds slowly to a change in input to $G_c(s)$ because of the large integral time constant. The transfer function $G_L(s)$ is chosen to match the response of the inferential control systems to the feedback control system. That is

$$G_L(s) = G_m(s)C(s)G_I(s)$$

To justify the above choice of $G_L(s)$, it is necessary to describe the response of the product quality to a change in set point. From Figure 4, the response of $y(s)$ to $y_d(s)$ is given by

$$y(s) = C(s)\{G_I(s)y_d(s) + G_c(s)[G_L(s)y_d(s) - G_m(s)y(s)]\}$$

Rearranging terms, we get

$$y(s) = [I + C(s)G_c(s)G_m(s)]^{-1} [C(s)G_I(s) + C(s)G_c(s)G_L(s)]$$

If

$$G_L(s) = G_m(s)C(s)G_I(s)$$

then

$$y(s) = [I + C(s)G_c(s)G_m(s)]^{-1} [I + C(s)G_c(s)G_m(s)]C(s)G_I(s)y_d = C(s)G_I(s)y_d(s)$$

Since $C(s)G_I(s) \approx I$, the response of the product quality to the set point will be rapid.

In some process control applications, it may be desirable to combine all three types of control systems: inferential, feedforward, and feedback. The block diagram configuration of such a control system is a linear combination of Figures 3 and 4.

THE DYNAMICS OF INFERENTIAL CONTROL SYSTEMS

The Laplace domain description of the process is taken to be (Figure 1)

$$y(s) = B^T(s)u(s) + C(s)m(s) \quad (1a)$$

$$\theta(s) = A^T(s)u(s) + P(s)m(s) \quad (1b)$$

The estimator $\alpha(s)$ satisfies the relationship

$$A(s)\alpha(s) = B(s) + R(s) \quad (2)$$

where $\alpha(0)$ is chosen to minimize the relative error as described in Part I. If there is no measurement noise, then $\alpha(0)$ minimizes $||R(0)||_2$. The details of the selection of $\alpha(s)$ for a particular example are given later. A general design procedure is given in Part III. For the purpose of the following discussion, $\alpha(s)$ is assumed to have already been selected.

The response of the process output to the unmeasured disturbances is denoted as $d(s)$, where

$$d(s) \equiv B^T(s)u(s) \quad (3)$$

The estimate of the effect of the unmeasured disturbances on the process output is denoted by $\hat{d}(s)$ and from Figure 1 is given by

$$\hat{d}(s) = \alpha^T(s)\ [\theta(s) - \hat{P}(s)m(s)] \quad (4)$$

From (1*b*), (2), and (3)

$$\hat{d}(s) = \alpha^T(s)\{A^T(s)u(s) + [P(s) - \hat{P}(s)]m(s)\}$$

$$= [B^T(s) + R^T(s)]u(s) + \alpha^T(s)[P(s) - \hat{P}(s)]m(s)$$

$$= d(s) + R^T(s)u(s) + \alpha^T(s)[P(s) - \hat{P}(s)]m(s) \quad (5)$$

Clearly, smaller values of $R(s)$ and $P(s) - \hat{P}(s)$ yield a better approximation of $\hat{d}(s)$ to $d(s)$.

The response of the control effort to a disturbance is given by

$$m(s) = G_I(s)\ [y_d(s) - \hat{d}(s)]$$

$$= G_I(s)y_d(s) - G_I(s)\ \{d(s) + R^T(s)u(s) + \alpha^T(s)[P(s) - \hat{P}(s)]m(s)\} \quad (6)$$

Rearranging, we get

$$m(s) = \{I + G_I(s)\alpha^T(s)[P(s) - \hat{P}(s)]\}^{-1}G_I(s)[y_d(s) - d(s) - R^T(s)u(s)] \quad (7)$$

Finally, the process output is given by

$$y(s) = C(s)m(s) + d(s)$$

$$= C(s)F(s)G_I(s)[y_d(s) - R^T(s)u(s)] + [I - C(s)F(s)G_I(s)]d(s) \quad (8)$$

where

$$F(s) \equiv \{I + G_I(s)\alpha^T(s)[P(s) - \hat{P}(s)]\}^{-1}$$

When $\hat{P}(s) = P(s)$ over the frequency range of the system, then $F(s) = I$, and the system is stable provided that the original process is stable and the controller $G_I(s)$ is stable. The appropriate choice for $G_I(s)$ in order to obtain a fast response to disturbances and set point changes is, according to Equation (8)

$$G_I(s) = F^{-1}(s)C^{-1}(s) \quad (9)$$

It will generally not be possible to implement (9) exactly because the elements of the transfer matrix $C(s)$ will be lags whose numerator polynomials are at least 1 deg. lower than their denominator polynomials. This means that $C^{-1}(s)$ will contain elements which will not be realizable, such as pure differentiators. Further, some of the elements of $C^{-1}(s)$ can be unstable, especially if one or more of the elements of $C(s)$ were nonminimum phase. Thus, in general, $G_I(s)$ will be a stable approximation to $[C(s)F(s)]^{-1}$. However, the steady state gains of $G_I(s)$ should be chosen so that

$$G_I(0)F(0)C(0) = I \quad (10)$$

If (10) is satisfied, then the steady state error in the output for a step disturbance, will be

$$y(\infty) - y_d = -R(0)u(0) \quad (11)$$

Since a small $||R(0)||_2$ is achieved through design of the estimator, the steady state error given by (11) will also be small. Thus, significant dynamic errors in $\alpha(s)$, as reflected in $R(s)$, need not be serious, since such errors will decay and will not affect the stability of the inferential control system. On the other hand, errors in $\hat{P}(s)$ do affect stability and are, therefore, more serious.

When $\hat{P}(s)$ is not equal to $P(s)$, the effect of the control effort on the estimate is fed back to the controller as shown by Equation (6). In order to simplify the discussion, $m(s)$ is assumed to be a scalar so that $G_I(s)$ and $\alpha^T(s)[P(s) - \hat{P}(s)]$ are both scalar transfer functions. The steady state gains of $G_I(s)$ and $\alpha(s)$ are fixed by the steady state design procedure. Thus, the feedback will be either positive or negative, depending on the gains or time constants of $P(s) - \hat{P}(s)$. If $\hat{P}(0) = P(0)$, then the nature of the feedback depends on the time constants of $P(s) - \hat{P}(s)$. For example, consider the case where $G_I(s)$ $\alpha(s) = K$ and $P(s) = 1/Ts + 1$, $\hat{P}(s) = 1/\hat{T}s + 1$. Then, $P(s) - \hat{P}(s) = (\hat{T} - T)s/(Ts + 1)(\hat{T}s + 1)$. If $\hat{T} > T$, then the control system will be stable, independent of the gain K, of $G_I(s)\alpha(s)$. If $\hat{T} < T$, then the system will be unstable for sufficiently high values of K. As

TABLE 1. STEADY STATE OPERATING CONDITIONS OF THE DISTILLATION COLUMN

Feed component	Feed composition	Distillate composition	Bottom's composition
Ethane	0.03	0.125	0.00
Propane	0.20	0.782	0.021
Butane	0.37	0.093	0.456
Pentane	0.35	0.00	0.458
Hexane	0.05	0.00	0.065

Feed stage: 8th stage from the reboiler
Feed flow rate: 7.71 mole/min., as a saturated liquid
Reflux ratio: 0.75
Column vapor flow: 7.26 mole/min
Column pressure: 1.72 MPa (250 lb/in.^{2} abs)

another example, consider the case where $P(s) = P(0)/Ts + 1$, $\hat{P}(s) = \hat{P}(0)/Ts + 1$, where $\hat{P}(0) \neq P(0)$. Again, $G(s)\alpha(s) = K$. In this case, the system will be stable for any value of K if $P(0) > \hat{P}(s)$ [we assume $P(0) > 0$]. If $\hat{P}(0) > P(0)$, then the system will be unstable for sufficiently large K. Clearly, one class of errors is to be preferred over another. The designer should determine, insofar as possible, which kind of errors are preferred and adjust the parameters in $\hat{P}(s)$ to assure negative feedback.

The problem of matching $\hat{P}(s)$ to $P(s)$ is more critical in those systems where the gains of $C(s)G_I(s)\alpha^T(s)$ are large. The gain of $C(s)$ can always be taken as unity by properly scaling $m(s)$. In this case, the steady state gain of $G_I(s)$ [that is, $C^{-1}(0)$] is also unity. A high gain in an element of $\alpha(s)$ implies that the disturbances $u(s)$ do not have much effect on the secondary measurement which is multiplied by the high gain [Equation (4)]. The high gain amplifies the small signal produced by the effect of $u(s)$ on $\theta(s)$. Unfortunately, the high gain also amplifies any noise which may be present in the measurement. An even more undesirable effect of a high estimator gain, however, is that it can cause the entire control system to become unstable when $\hat{P}(s) \neq P(s)$. To avoid such problems, the designer should use the relative error criterion described in Part I along with a reasonable estimate of the covariance of the measurement noise. In those cases, where there is no measurement noise, it may be convenient to assume the presence of noise to hold down the estimator gains.

Another source of stability problems may arise if the gains in $P(s)$ are high. A high gain in $P(s)$ implies that the control effort affects the secondary measurement much more than it affects the product quality. In this situation, it becomes critically important that either $\hat{P}(s)$ be a good approximation to $P(s)$ and/or that the gains of the estimator be relatively low. If neither of the above can be achieved, then the designer should consider an alternate choice of control effort.

EXAMPLES OF INFERENTIAL CONTROL

The Process

Our example process is the same as that used by Weber (1972). It is a sixteen stage, five component distillation column with a total condenser and total reboiler. The column operating conditions are given in Table 1. As Weber did, we assume perfect mixing on each plate, adiabatic operation, constant molar overflow, and no hydraulic lag. The linearized column model was obtained from step tests on a simulation of the full nonlinear model. These data were fit by a first-order lag for each transfer function (Tong, 1974). The various parameters of the model are given in Tables 2 and 3. As in the previous section, the form of the linear model is

$$y(s) = B^T(s)u(s) + C(s)m(s)$$
$$\theta(s) = A^T(s)u(s) + P(s)m(s) \quad (1)$$

where $y(s) = [y_1(s), y_2(s)]^T$, $u(s) = [u_1(s), \ldots u_5(s)]^T$, and $\theta(s) = (\theta_1{}^1, \theta_2{}^3, \theta_3{}^8, \theta_4{}^{14}, \theta_5{}^{16})^T$; the superscript on θ denotes the stage number of the measurement. The subscript denotes the element number in the vector θ. Stages are numbered sequentially from the bottom of the column to the top.

In order to facilitate comparison of this work with that of Weber (1972), we restrict ourselves to the use of subsets of the measurements selected by him. The control objectives are the same as his: the control of the overhead butane composition and bottoms propane composition.

Overhead Butane Composition Control

Design of an Inferential Control System. The design of an inferential control system starts with the selection of secondary measurements which will be used to infer the product quality, in this case the overhead butane composition. The selection procedure (see Part I) aims at minimizing the number of measurements necessary to obtain accurate estimates which are insensitive to modeling errors. To estimate the overhead butane composition, only one temperature measurement is required, as can be seen from Figure 5. Figure 5 is a plot of the projection error in the overhead butane composition vs. the stage number of the temperature measurement. A small projection error implies that the inferential control system will exhibit only a small fraction of the steady state error which would have occurred without control. From Figure 5, the projection error is minimized by selecting stage 14 as the stage on which to measure the temperature. With this choice, the projection error is only 0.007 when the steady state gain of the estimator $\alpha(0)$ is chosen as 0.0045 (see Part I). Note that temperature 14 lies in Weber's measurement set.

TABLE 2. TRANSFER FUNCTIONS $b(s)$, $c_1(s)$, AND $c_2(s)$ FOR THE OVERHEAD BUTANE AND BOTTOM'S PROPANE COMPOSITIONS

y_1 = overhead butane composition
y_2 = bottom's propane composition

y_i	$\hat{b}_{i1}(s)$	$\hat{b}_{i2}(s)$	$\hat{b}_{i3}(s)$	$\hat{b}_{i4}(s)$	$\hat{b}_{i5}(s)$	$\hat{c}_{i1}(s)$	$\hat{c}_{i2}(s)$
y_1	$\frac{-0.188}{72s+1}$	$\frac{-0.163}{72s+1}$	$\frac{0.0199}{70s+1}$	$\frac{0.0043}{80s+1}$	$\frac{0.002}{85s+1}$	$\frac{-0.173}{70s+1}$	$\frac{0.0305}{75s+1}$
y_2	$\frac{0.0174}{15s+1}$	$\frac{0.0259}{13s+1}$	$\frac{0.0045}{4s+1}$	$\frac{-0.00029}{3s+1}$	$\frac{-0.00099}{3s+1}$	$\frac{0.015}{18s+1}$	$\frac{-0.00768}{7s+1}$

TABLE 3. TRANSFER FUNCTIONS $a(s)$ AND $p(s)$ FOR THE TEMPERATURES ON PLATES 1, 3, 8, 14, AND 16

Stage No.	Var. (i)	$\hat{a}_{i1}(s)$	$\hat{a}_{i2}(s)$	$\hat{a}_{i3}(s)$	$\hat{a}_{i4}(s)$	$\hat{a}_{i5}(s)$	$\hat{p}_{i1}(s)$	$\hat{p}_{i2}(s)$
1	1	$\frac{-7.99}{9s+1}$	$\frac{-9.78}{9s+1}$	$\frac{-5.28}{5s+1}$	$\frac{3.59}{8s+1}$	$\frac{6.09}{5s+1}$	$\frac{7.47}{8s+1}$	$\frac{2.70}{4s+1}$
3	2	$\frac{-11.29}{12s+1}$	$\frac{-15.91}{12s+1}$	$\frac{-4.23}{5s+1}$	$\frac{3.63}{8s+1}$	$\frac{4.75}{5s+1}$	$\frac{9.80}{15s+1}$	$\frac{3.79}{5s+1}$
8	3	$\frac{-18.28}{5s+1}$	$\frac{-16.43}{10s+1}$	$\frac{-0.47}{5s+1}$	$\frac{3.96}{3s+1}$	$\frac{4.60}{1.5s+1}$	$\frac{8.20}{30s+1}$	$\frac{2.30}{18s+1}$
14	4	$\frac{-42.02}{50s+1}$	$\frac{-35.92}{70s+1}$	$\frac{4.45}{65s+1}$	$\frac{1.10}{70s+1}$	$\frac{0.46}{75s+1}$	$\frac{36.0}{65s+1}$	$\frac{6.82}{70s+1}$
16	5	$\frac{-50.47}{25s+1}$	$\frac{-25.26}{75s+1}$	$\frac{3.15}{70s+1}$	$\frac{0.68}{78s+1}$	$\frac{0.32}{80s+1}$	$\frac{30.0}{67s+1}$	$\frac{3.46}{70s+1}$

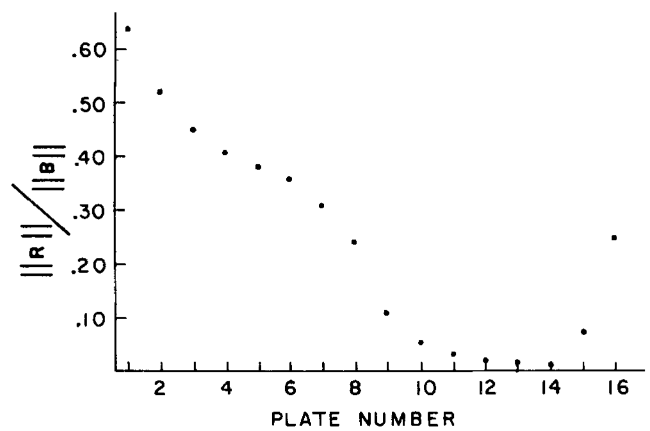

Fig. 5. Projection error in overhead butane composition for single temperature measurement.

Since there is only a single measurement, the estimator $\alpha(s)$ is a scalar transfer function.

The dynamic elements of the estimator $\alpha(s)$ (see Figure 1) are selected to match the dynamic response of the estimate to the actual product composition. From Equation (2), the dynamic error in estimation $e(s)$ is given by

$$e(s) = [b_1^T(s) - a_4^T(s)\alpha(s)]u(s) \equiv r^T(s)u(s) \qquad (12)$$

where $r(s) = b_1(s) - a_4^T(s)\alpha(s)$.

$b_1(s)$ and $a_4(s)$ are the vectors which relate the response of the overhead butane product composition and the temperature on stage 14 to the disturbances. The disturbances are the feed component flow rates. Approximations to $b_1(s)$ and $a_4(s)$ are found in Tables 2 and 3.

If $r(s)$ in Equation (12) is zero, then the estimate is perfect. In general, and for this problem in particular, there is no choice of estimator which will yield perfect dynamic estimates. It is possible to select $\alpha(s)$ to minimize the expected error in the estimates (see Part III); however, such a choice for $\alpha(s)$ generally leads to an unnecessarily complex estimator. In this paper, we arbitrarily restrict our dynamic compensation to be lead-lag networks. The problem then is to choose the lead and lag time constants to best match the dynamic response of the estimate with the actual process. Again, we defer a precise mathematical treatment of the selection procedure to Part III and adopt an ad hoc strategy for selecting the lead and lag time constants which is based on the following considerations.

The transfer functions in the vectors $b_1(s)$ and $a_4(s)$ are simple lag networks. If all of the lags in $b_1(s)$ and $a_4(s)$ were the same (but not necessarily equal to each other), then $r(s)$ could be made zero by choosing $\alpha(s)$ as a lead-lag network with a lead time constant equal to the lag time constant in $a_4(s)$ and a lag time constant equal to the lag time constant in $b_1(s)$. Actually, the lag time constants in $b_1(s)$ range from 70 to 85 min, while the lag time constants in $a_4(s)$ range from 50 to 70 min (see Tables 2 and 3). Since the above range of time constants is not large, it seems reasonable to choose the lead time constant in the estimator as the average of the time constants in $a_4(s)$. Similarly, the lag time constant in $\alpha(s)$ is chosen as the average of the time constants in $b_1(s)$. That is

$$\alpha(s) = 0.0045 \frac{(66s + 1)}{(77s + 1)} \qquad (13)$$

Having designed the estimator $\alpha(s)$, we now have to select the controller $G_I(s)$ and the control effort compensation $\hat{P}(s)$ shown in Figure 1. Again, because there is but a single measured temperature, $\hat{P}(s)$ is a scalar transfer function. This transfer function is an approximation of the response of the temperature on stage 14 to changes in the control effort (that is, the reflux flow), and it is given by $\hat{p}_{41}(s)$ as listed in Table 3:

$$\hat{P}(s) = \hat{p}_{41}(s) = \frac{36}{65s + 1} \qquad (14)$$

For perfect control, given a perfect estimate, the controller transfer function $G_I(s)$ should be the inverse of the transfer function between the overhead butane composition and the reflux flow. From the data in Table 2, this controller would have proportional plus pure derivative actions. However, since the estimate is not perfect and the measurements are always corrupted by noise, a pure derivative action is inappropriate because it amplifies such errors. As is common in such situations, the pure derivative action is replaced by a lead network, and $G_I(s)$ is taken as

$$G_I(s) = -5.78 \left(\frac{70s + 1}{10s + 1} \right) \qquad (15)$$

The inferential control system given by (13), (14),

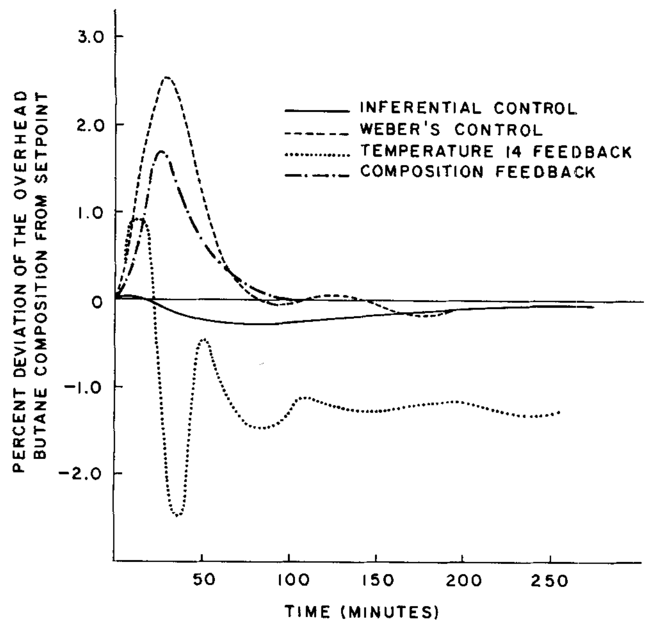

Fig. 6. Overhead butane composition response to 10% step change in the butane feed flow rate.

and (15) can be implemented inexpensively with either digital or analogue hardware. All that is required is the measurement of the temperature on stage 14 and the reflux flow. The entire control system consists of only three lead-lag networks.

The above inferential control system was tested by using it to control the overhead butane composition of the simulated distillation column for various feed disturbances. The simulation of the nonlinear column dynamics is described by Weber (1971). Figure 6 shows the response of the column under inferential control for a step change in the butane flow in the feed. That is, the butane component flow rate increases, while all other component flows remain fixed. Such a change means that all feed compositions are changed. Changes in other feed component flow rates yield responses similar to that shown in Figure 6 (Tong, 1974).

The inferential control system design strategies presented above and in Parts I and III are extensions of the design strategies developed by Weber (1972). His design method differs from those presented here mainly in that his estimator contains no dynamic compensation, and his estimator needs as many measurements as there are unknown disturbances. Weber's system for control of the overhead butane composition can be rearranged to fit into the form of Figure 1, with the following transfer functions:

$$G_I(s) = \frac{G_c(s)}{1 + C(s)G_c(s)}; \quad G_c(s) = K\left(1 + \frac{1}{\tau_I s}\right) \tag{16a}$$

$$\alpha^T(s) = 10^{-4}(0.127, \ -1.29, \ -0.76, \ 45.0, \ -0.21) \tag{16b}$$

$$\hat{P}^T(s) = (7.47, \ 9.80, \ 8.20, \ 36.0, \ 30.0) \tag{16c}$$

The gain K and integral time constant τ_I of $G_c(s)$ were chosen so that $G(s) \approx [\hat{C}(s)]^{-1}$ over the frequency range of interest. Therefore, the controller $G_I(s)$ is virtually the same in Weber's design and the current design. Weber's estimator, Equation (16*b*), is also very similar to the estimator of the current design because even though there are five measurements, the weight associated with the temperature on the stage 14 (that is, 45), is so much greater than the other weights that effectively only temperature 14 is being used. The significant difference between Weber's system and the current system lies in the lack of dynamic compensation for $\hat{P}(s)$ [see Equation (16*c*)]. As can be seen from Figure 6, the dynamic lag included in the control effort compensation $\hat{P}(s)$ of the current work significantly improves the dynamic behavior of the inferential control system.

Alternate Control Systems. Figure 6 allows comparisons between the response of inferential control systems and two standard control systems. The curve labeled composition feedback was obtained with the column under control of a tuned proportional plus integral controller which utilizes instantaneous measurements of the overhead butane composition. The curve labeled temperature 14 feedback is the response of the product composition when a tuned proportional plus integral controller is used to maintain a constant temperature on stage 14.

The fact that the inferential control system performs significantly better than the temperature feedback control system, when both systems use the same secondary measurement (that is, the temperature on stage 14), deserves explanation. The block diagram for the temperature feedback control system is given in Figure 7. For the system of Figure 7, the response of the product composition $y(s)$ to an input disturbance $u(s)$ is given by

$$y(s) = \left[B^T(s) - \frac{G_c(s)C(s)A^T(s)}{1 + P(s)G_c(s)}\right]u(s) \tag{17}$$

Normal control system design procedures lead to a control system with a unity gain between the set point θ_d and the output $\theta(s)$ for low frequencies. This means that

$$\frac{G_c(s)C(s)}{1 + P(s)G_c(s)} \approx \frac{C(s)}{P(s)} \quad \text{for low frequencies} \tag{18}$$

At steady state, therefore

$$y(0) = \left[B^T(0) - \frac{C(0)A^T(0)}{P(0)}\right]u(0) \tag{19}$$

The conditions required for (19) to be zero independent of $u(0)$ are more stringent than those required for a single temperature inferential control system. First, the vector $A(0)$ associated with the chosen temperature measurement must be parallel to the vector $B(0)$. That is, the disturbances must affect the temperature in the same way that they affect the product composition. Second, the ratio of $C(0)$ to $P(0)$ must be such that $A(0)$ is scaled to the same length as $B(0)$. This latter requirement is the same as requiring $C(0)/P(0) = \alpha(0)$, as can be seen from Equation (2) with $R(0) = 0$. The first condition is also required for perfect estimation in an inferential control system. However, the second condition is not required by

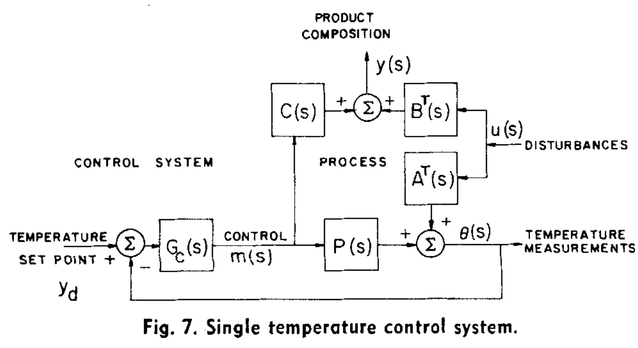

Fig. 7. Single temperature control system.

the inferential system. Thus, the steady state behavior of a single temperature feedback control system will generally not be as good as that of a single temperature inferential control system. The inferential control system is also likely to be less oscillatory than the feedback system. This is true because the inferential system will have the same stability properties as the process, provided that $P(s) \cong \hat{P}(s)$, while the feedback system will generally be designed to decrease the process stability in order to speed up the process response.

Implementation. The simplest and least expensive of the above control systems is the temperature feedback controller. This system requires only a PI controller which can be easily implemented with digital or analogue control hardware. The inferential control system is only slightly more complex than the constant temperature controller. It requires the implementation of two lead-lag networks in addition to a controller which is also a lead-lag network. The inferential controller also requires measurement of the reflux flow rate. However, this measurement is available from the reflux flow controller which is usually installed as a first level controller in all of the various control systems. Again, the required lead-lag networks can be easily and inexpensively implemented with either analogue or digital hardware. Most expensive of the control systems, in terms of hardware, is the composition feedback controller. This system usually requires an on-line chromatograph which is often cascaded with a temperature feedback control system.

In terms of engineering effort required for implementation, the inferential control system will probably be the most expensive, at least in the initial applications. A great deal of engineering effort has already been expended in selecting the appropriate stage for temperature control (Boyd, 1975; Luyben, 1969; Wood, 1967) and in making chromatographs reliable. Fortunately, however, excellent steady state models are available for most distillations, and the critical dynamic models [that is, $\hat{P}(s)$] are readily obtainable in the field.

Bottoms Propane Composition Control

The purpose of this example is to show the behavior of a single product inferential controller when many measurements are used in carrying out the inference of product quality. As can be seen from Figure 8, there is no single temperature which can be used to infer the bottoms propane composition exactly. Of the five measurements available, use of the temperature on stage 3 results in the minimum projection error. Figure 9 shows that the single temperature inferential controller using the temperature on stage 3 exhibits a significant steady state offset, as expected. The estimator and controller for this control system are

$$\alpha(s) = -0.0014\frac{8.4s+1}{7.6s+1} \tag{20a}$$

$$\hat{P}(s) = \frac{3.79}{5s+1} \tag{20b}$$

$$G_I(s) = -\frac{(130(7s+1)}{(0.7s+1)} \tag{20c}$$

The control effort is the vapor boilup rate.

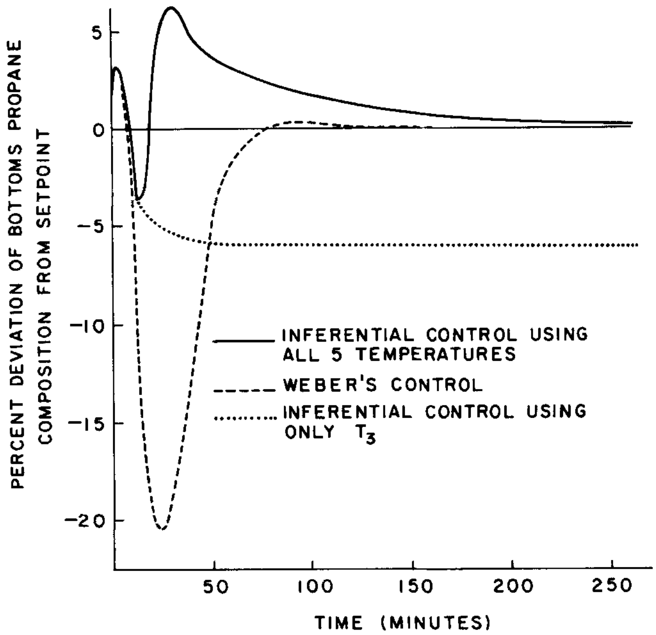

Fig. 9. Bottom's propane composition response to 10% step change in the butane feed flow rate.

To avoid the steady state offset exhibited by the single temperature inferential controller, additional temperature measurements are needed. Probably one additional measurement would suffice. However, for the sake of illustration, we will use all five available measurements. The estimator $\alpha(s)$ is then given by

$$\alpha(s) = \frac{10^{-4}[-0.99(7.2s+1),\ -17.6(8.4s+1),\ 18.2(4.9s+1),\ -7.99(66s+1),\ 0.69(66s+1)]^T}{(7.6s+1)} \tag{21}$$

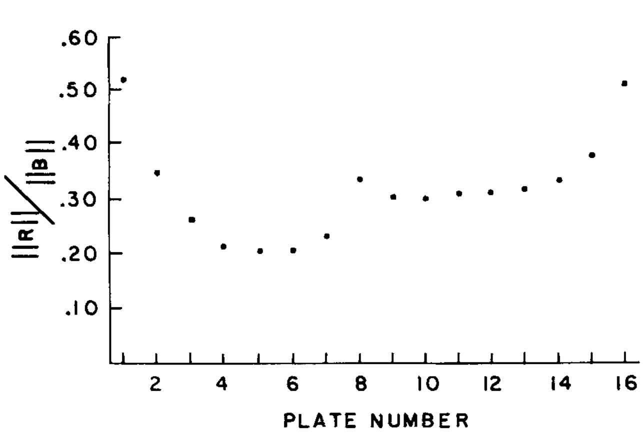

Fig. 8. Projection error in bottoms propane composition for single temperature measurement.

The numerator time constants in (21) were obtained by averaging the time constants for the response of each temperature to the various disturbances. The denominator time constant of 7.6 min is the average of the time constants for the response of the bottoms propane composition to the disturbances. The transfer functions for $\hat{P}(s)$ are obtained from Table 3. The controller $G_I(s)$ is the same as that given by (20c).

The response of the five measurement inferential control system to a feed composition disturbance is shown in Figure 9. The initial cycling exhibited by the control system is probably due to the mismatch between $P(s)$ and $\hat{P}(s)$. Also shown in Figure 9 is the response of Weber's inferential control system. Again, Weber's system is the same as ours except that $\alpha(s)$ and $\hat{P}(s)$ are replaced by constant vectors [that is, set $s = 0$ in (21) and Table 3].

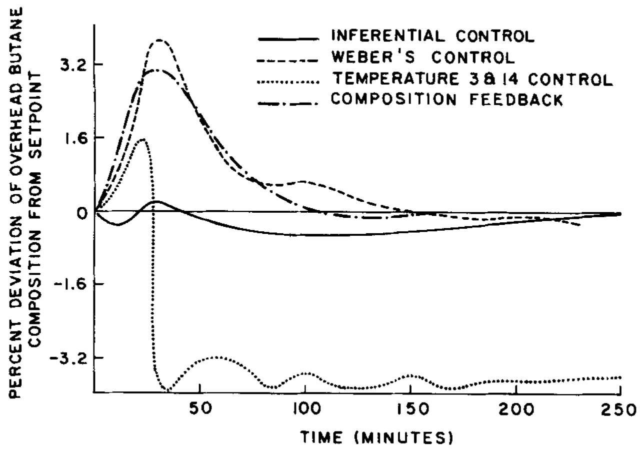

Fig. 10. Overhead butane composition response to 10% change in butane feed flow rate under dual control.

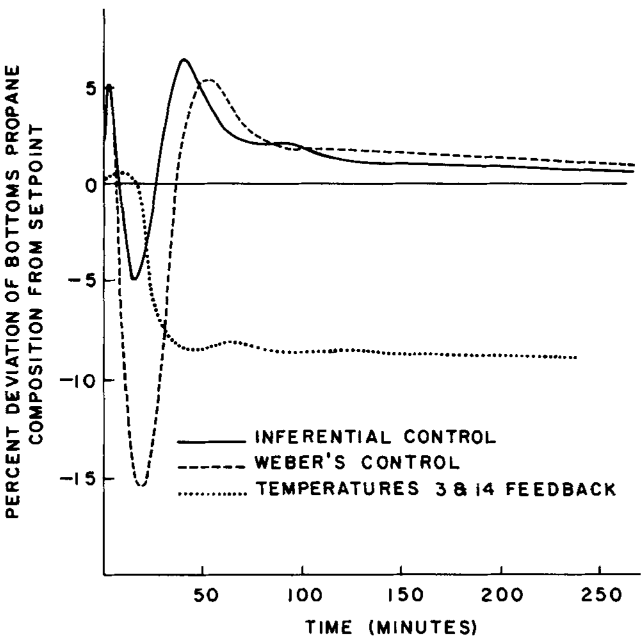

Fig. 11. Bottom's propane composition response to 10% step change in the butane feed flow rate under dual control.

Simultaneous Overhead and Bottoms Product Control

A significant advantage of inferential control over feedback control is that multiple product control does not suffer from the stability problems which arise in feedback control due to control loop interactions. Provided that the compensation $\hat{P}(s)$ has been properly selected, the multivariable inferential control system responds to disturbances in much the same way as a feedforward control system.

To illustrate the application of multivariable inferential control, we shall combine the previous two control systems so as to achieve simultaneous control of the overhead butane and bottoms propane compositions of our simulated multicomponent distillation column. As before, the control efforts are taken as the reflux flow and vapor boilup rates.

The estimator for the two product control system is simply the combination of the two previous estimators. Mathematically, $\alpha(s)$ is now a 5×2 matrix where the first column of $\alpha(s)$ contains a single nonzero element in position 4 which is given by (13). The second column in $\alpha(s)$ is given by (21).

The control effort compensation $\hat{P}(s)$ for the two product inferential control system is a 5×2 matrix of transfer functions. The second column of the matrix is the same as the compensation for the bottoms propane inferential controller. However. the first column of $\hat{P}(s)$ differs from the compensation for the overhead butane controller which was only a scalar transfer function. The difference arises from the fact that all of the temperatures are influenced by a change in reflux flow rate and, therefore, this influence must be subtracted from each of the five temperature measurements. The transfer functions for $\hat{P}(s)$ are given in the last two columns of Table 3.

The controller matrix $G_I(s)$ of Figure 1 is the approximate inverse of the matrix $\hat{C}(s)$ which relates the control effort to the product compositions [see Equation (1)]. The inverse of $\hat{C}(s)$ is given by

$$[\hat{C}(s)]^{-1} \equiv K(s) \equiv [k_{ij}(s)] \tag{22}$$

$$k_{11}(s) = -8.8.7\,\frac{(70s+1)(75s+1)(18s+1)}{(78s+1)(23s+1)}$$

$$k_{12}(s) = -35.01\,\frac{(70s+1)(18s+1)(7s+1)}{(78s+1)(23s+1)}$$

$$k_{21}(s) = -17.22\,\frac{(70s+1)(75s+1)(7s+1)}{(78s+1)(23s+1)}$$

$$k_{22}(s) = -198.6\,\frac{(75s+1)(18s+1)(7s+1)}{(78s+1)(23s+1)}$$

The components of $G_I(s)$ are implemented as $k_{ij}(s)/(2.5s+1)$ to avoid differentiation. In practice, each term in $G_I(s)$ can probably be approximated as a first-order lead network.

As can be seen from the above design, the implementation of the multivariable inferential control system is somewhat more complex than the sum of the implementations of the single variable inferential control systems. The added complexity arises from the fact that all secondary measurements must be compensated for all control efforts and from the requirement that the controller approximate an inverse matrix of transfer functions $C^{-1}(s)$. In our example, the above requirements resulted in the implementation of four additional transfer functions in $\hat{P}(s)$ and two additional transfer functions in $G_I(s)$.

The response of the above two product inferential control system to previously used disturbances is shown in Figures 10 and 11. The initial cycling of the inferential control system is probably due to a mismatch between $\hat{P}(s)$ and $P(s)$. For the purpose of comparison, Figures 10 and 11 also show the response of two single loop feedback controllers which attempt to simultaneously maintain temperatures on stages 3 and 14 at their respective set points. The single loop temperature controllers were turned so as to minimize control loop interactions. Nonetheless, a small degree of interaction is evident from the response of the overhead composition while under temperature control as shown in Figure 10. The temperature control systems also compare poorly to the inferential control system in terms of maintaining the desired steady state product compositions.

ACKNOWLEDGMENT

This work was supported by NSF grant GK 35935 and the Industrial Sponsors of the Control of Complex Systems group of the Systems Research Center at Case Western Reserve University.

NOTATION

$A(s)$, $B(s)$, $C(s)$, $P(s)$ = process transfer function matrices
$P(s)$, $Q(s)$ = process transfer function matrices (see Figures 1 and 2)
$a_i(s)$ = i^{th} column of $A(s)$
$b_i(s)$ = i^{th} column of $B(s)$
$d(s)$ = effect of the disturbances on the product qualities
$F(s)$ = $\{I + \alpha^T(s)\,[P(s) - \hat{P}(s))]\}^{-1}$
$G_I(s)$ = matrix of controller transfer functions for the inferential control loop
$G_c(s)$ = matrix of controller transfer functions for the feedback control loop
$G_m(s)$ = matrix of measurement lags
$G_L(s)$ = matrix of compensators to match set point changes
$m(s)$ = control effort vector
$m_1(s)$ = reflux ratio
$m_2(s)$ = vapor boilup rate
$R(s)$ = $A(s)\alpha(s) - B(s)$
$r_j(s)$ = $b(s) - a_j^T(s)\alpha(s)$, vector of residuals associated with the j^{th} temperature measurement
s = Laplace transform variable
$u(o)$ = steady state value of disturbance
$u(s)$ = unmeasured disturbance vector
$u_i(s)$ = flow rate of i^{th} component in feed
$v(s)$ = measured disturbance vector
$y(s)$ = product quality vector
$y_1(s)$ = overhead butane composition
$y_2(s)$ = bottoms propane composition
$\alpha(s)$ = matrix of estimator transfer functions
$\theta(s)$ = measured output vector

Superscripts

$\wedge$ = estimated value
-1 = inverse
T = transpose

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Manuscript received February 2, 1977; revision received January 3, and accepted January 6, 1978.

Part III. Construction of Optimal and Suboptimal Dynamic Estimators

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Methods are presented for the construction of optimal and suboptimal estimators for inferential control systems. Optimal estimators are constructed with the aid of Kalman filtering techniques applied to linear systems driven by integrated white noise disturbances. The description of the disturbances as integrated white noise leads to optimal dynamic estimators which reduce to optimal static estimators for sustained disturbances. Suboptimal estimators are constructed by prespecifying the structure of the estimator and choosing estimator parameters so as to minimize the mean square error in estimation.

Optimal and suboptimal estimators are compared by using them in an inferential control system which attempts to control the product composition of a simulated multicomponent distillation column. There is little difference in performance between optimal and suboptimal estimators which use temperature and flow measurements to estimate product composition. However, the inferential control system using a simple suboptimal estimator is significantly superior to the policy of maintaining a selected tray temperature constant through a standard feedback control system. The inferential control system is also superior to composition feedback control systems where the measurement delay is greater than 5 min.

Part I discusses the problem of measurement selection and estimator design to achieve optimal steady state behavior. Part II discusses the dynamic behavior of inferential control systems based on ad hoc methods for adding dynamic compensation to the estimator. This work is concerned with the design of optimal and nearly optimal dynamic compensation for the estimator.

An important feature of the design methods is that they yield linear dynamic estimators which reduce to optimal static estimators in the steady state. This is achieved by using a statistical description of the input disturbances which admits the possibility of the process

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